

Space Plasma Physics Fall 2015

Problem Set 2

Due date: Nov. 18, 2016

1. MHD power generators may possibly be a more efficient way of converting heat into electricity. Think of one as consisting of a simple rectangular channel of (x-) width, a , (y-) height b , in which the plasma flows under pressure in the z -direction. Take the plasma density and velocity to be uniform. A uniform magnetic field, B , is applied in the y -direction and the walls at $x = 0, a$ are electrodes where the electric current density (density j , assumed uniform) is picked off at a voltage difference Φ . Use the MHD equations to answer the following questions:

- (a) If the resistivity, η , of the plasma is negligible, what is the plasma velocity?
- (b) If the pressure is P_0 at $z = 0$, what is its value as a function of z ?
- (c) How much electric power is generated per unit length of the channel?
- (d) What is the rate of doing work per unit channel length by the plasma pressure force?
- (e) If η is not negligible but can be considered fixed, and the flow velocity and B -field are also fixed but the current density can be varied, what is the maximum electric power unit length that can be generated?

2. A θ - pinch (By symmetry, B has only z -component, j has only θ component and ∇p has only r component, so called because plasma currents flow in θ direction,) in MHD equilibrium has magnetic field that is

$$\mathbf{B}(r) = B_0 + (B_a - B_0)r/a$$

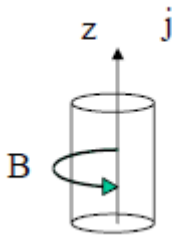
for $0 \ll r \ll a$

where the plasma edge is $r = a$, at which point the plasma pressure, p , is zero. Calculate:

- (a) The pressure profile, $p(r)$
- (b) The current density profile, $j(r)$
- (c) The maximum possible value of the β , $2\mu_0 \langle p \rangle / B_a^2$, where $\langle p \rangle$ is the volume average plasma pressure:

$$\langle p \rangle = \int_0^a p 2\pi r dr / \pi a^2$$

3. Z-pinch



So called because j follows in z -direction.

(a) For a z -pinch equilibrium which has zero plasma pressure at the plasma edge, $r = a$, prove by integrating the MHD force balance equation a second time that the volume-averaged pressure is a function only of the total current, and find the function.

(b) If a hydrogen plasma z -pinch has uniform density $n = 10^{20} \text{ cm}^{-3}$, temperature $T_e = T_i = T_0(1 - r^2/a^2)$ with $T_0 = 10 \text{ keV}$, and radius $a = 0.01 \text{ m}$, what current is required?