Space Plasma Physics Fall 2017

Problem Set 5

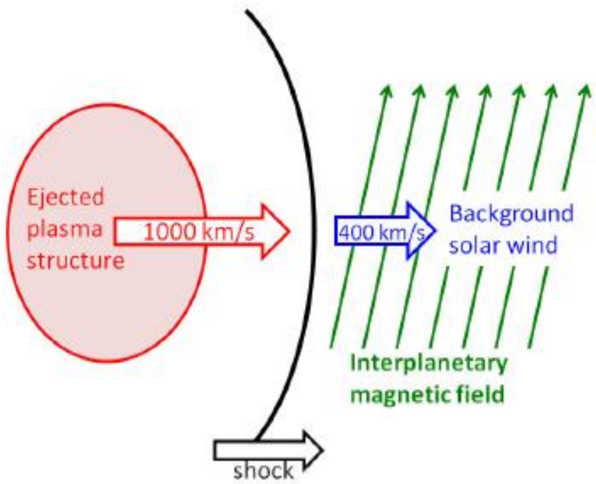
Due date: Dec. 29, 2017

1. From the energy conservation equation as derived in Problem Set 1, please show the shock – jump condition is:

$$[\rho u_n\left(\frac{1}{2}u^2+\frac{\gamma}{\gamma-1}\frac{p}{\rho}\right)+u_n\frac{B^2}{\mu_0}-\boldsymbol{u}\cdot\boldsymbol{B}\frac{B_n}{\mu_0}]=0$$

2. To obtain the boundary conditions at the discontinuity surface we used conservation of mass, momentum and energy, but not entropy. (a) Consider an ideal gas without magnetic field, show the density ratio $\mathrm{r}=\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)p_2+(\gamma-1)p_1}{(\gamma-1)p_2+(\gamma+1)p_1}$; (b) The entropy of an ideal gas with constant specific heats is given by $S=C_v\ln\left(\frac{p}{\rho^\gamma}\right)$, please calculate the difference between the entropy on each side of the shock front and show that the entropy jump increases with strength of the shock wave (but is entirely independent of the dissipative mechanism).

3. Due to the presence of hot corona, solar wind fills up the interplanetary space. Usually, the solar wind speed is about 400 km/s, temperature is about 10^5 K, the number density of protons is about 5 cm^{-3}, and the magnetic field strength is about 10 nT. An ejection occurs in the solar corona. It ejected a huge plasma structure into the interplanetary space with a speed of 1000 km/s, which continuously drives a shock ahead of it as shown as follows:



(a) Which side is the upstream?
(b) Which type of the driven shock is, fast shock or slow shock?
(c) If the interplanetary magnetic field is perpendicular to the shock normal, derive the compression ratio of the shock.