# Space Plasma Physics Fall 2018

# Problem Set 6

**Due date: Jan. 4, 2018**

1. (a) Derive the dispersion relation for a two-stream instability occurring when there are two cold electron streams which equal and opposite $\mathbf{v}_0$ in a background of fixed ions. Each stream has a density $\frac{1}{2}n_0$

(b) Calculate the maximum growth rate.

2. Interchange instability and volume per unit flux: Another way to consider interchange instability is to calculate the consequence of interchanging two flux tubes having the same magnetic flux. Doing so will not change the magnetic energy since the magnetic field is unchanged by this interchange. However, if the flux tubes contain finite-pressure plasma and the volumes of the two flux tubes differ, the interchange will result in compression of the plasma in the flux tube which initially had the larger volume and expansion of plasma in the flux tube which initially had the smaller volume. The former require work on the plasma and the latter involves work by the plasma. If the net work must be done on the plasma to affect the interchange, then the interchange is stable and vice versa. In a magnetic confinement confinement configuration, the high pressure is by assumption on the inside of the configuration and the low pressure is on the outside. Thus, the question is whether interchange a high pressure, inner region flux tube with a low pressure out region flux tube requires positive or negative work.

(a) Show that the volume per unit flux in a flux tube is given by

$$V' = \int \frac{dl}{B}$$

where the contour is over the length of the flux tube.

(b) Show that instability corresponds to $V'$ increasing on going outwards.

(c) Consider the magnetic field external to a current-carrying straight wire. How does $V'$ scale with distance from the wire and would a plasma confined by such a magnetic field be stable or unstable to interchanges?

3. The sausage instability can be inhibited by a longitudinal magnetic field applied inside the plasma column. This longitudinal magnetic field can be produced by passing a current through a solenoidal coil wound around the column. Please explain this physically.

Next we shall determine what must be the magnitude Bz of the longitudinal magnetic flux density, as compared to the magnitude of the azimuthal Bϴ field, in order to that the longitudinal field be able to stabilize the plasma column against the setting of the sausage instability.

（a）If the radius r of the column, at the magnetic field, at the constriction, is decreased by an amount dr, and considering that the magnetic flux ($\Phi_m = B_z \pi r^2$) through the cross-sectional area of the column remains constant during compression. Show the corresponding internal magnetic pressure increases by $dp_z = -\frac{2B_z^2}{\mu_0}\frac{dr}{r}$

(b) Show the corresponding increase in the in the external Bϴ magnetic field is $dp_\theta = -\frac{B_\theta^2}{\mu_0}\frac{dr}{r}$

(c) Therefore, in order the plasma column be stable against the sausage distortion, we must have $B_z^2 > \frac{1}{2}B_\theta^2$