

An Analytical Model to Predict the Arrival Time of Interplanetary CMEs

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Abstract Referring to the aerodynamic drag force, we present an analytical model to predict the arrival time of coronal mass ejections (CMEs). All related calculations are based on the expression for the deceleration of fast CMEs in the interplanetary medium (ICMEs), $\dot{v} = -\frac{1}{15700}(v - V_{\text{SW}})^2$, where V_{SW} is the solar wind speed. The results can reproduce well the observations of three typical parameters: the initial speed of the CME, the speed of the ICME at 1 AU and the transit time. Our simple model reveals that the drag acceleration should be really the essential feature of the interplanetary motion of CMEs, as suggested by Vršnak and Gopalswamy (*J. Geophys. Res.* **107**, 1019, 2002).

Keywords Coronal mass ejections (CMEs)

1. Introduction

Coronal mass ejections (CMEs) are found to be the primary cause of severe geomagnetic storms (Gosling, 1993). The prediction of their arrival times at Earth is desirable in space weather researches. Gopalswamy *et al.* (2000, 2001) proposed an empirical model of the acceleration/deceleration of CMEs using their initial speeds observed with a coronagraph. The shock-time-of-arrival model (Dryer and Smart, 1984; Smart and Shea, 1985) assumes that an interplanetary shock propagates explosively like a supernova explosion and predicts its arrival using the velocity of the disturbance within the corona determined from observation of type II solar radio bursts at metric wavelengths. By combining the observation of interplanetary scintillations (I), the dynamics of solar wind storm propagation (S) and fuzzy mathematics (F), Wei and his coworkers (Wei and Cai, 1990; Wei, Xu, and Feng, 2002; Wei, Cai, and Feng, 2003; Wei *et al.*, 2005) presented an ISF prediction method for the geomagnetic disturbances. Smith and Dryer (1990) devised a 2.5D MHD simulation model, called the interplanetary shock propagation model, in which the

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net energy input into the solar wind is the main organizing parameter. It is very interesting that recently Feng *et al.* (2009a) creatively developed a database method for predicting the shock arrival time with the purpose of providing an operational method for all space weather events during solar cycle 23. They created databases using previous numerical prediction models by considering the effects of the initial shock speed and the source longitude. Their prediction tests show that the database method is powerful and very promising when applied to space weather events of other solar cycles. Vršnak and Gopalswamy (2002) gave a semi-empirical model for estimating the transit time on the assumption that the only force acting upon the CME in interplanetary space (ICME) is the aerodynamic drag. There are also several other drag-based papers (*e.g.*, Cargill, 2004; Vršnak and Žic, 2007; Vršnak, Vrbanc, and Čalogović, 2008; Borgazzi *et al.*, 2009) which discuss the relations between the transit time and the solar wind speed, solar wind density, CMEs' mass, and other factors. Here, by simplifying the coefficient γ in Vršnak and Gopalswamy (2002), and using different events, we reargue whether it is a good way to assume the drag acceleration for the interplanetary motion of CMEs.

2. Event Selection

All of our collected 60 CME–ICME pairs come from the list of Richardson/Cane ICMEs in 1996–2007 (Richardson and Cane, 2007). Its related web site is <http://www.ssg.sr.unh.edu/mag/ace/ACElists/ICMEtable.html>. As listed in Table 1, our events should still satisfy three conditions.

- (1) The CME must be the halo one. Such an item may better ensure that it belongs to an earth-directed CME and the distance it traveled is close to 1 AU.
- (2) The difference between the linear speed (V'_{CME}) and the speed obtained from a quadratic fit (V_{CME}) at $20R_s$ (R_s is the solar radius) should be less than 20%. V'_{CME} and V_{CME} are both obtained from the SOHO LASCO CME catalog (see http://cdaw.gsfc.nasa.gov/CME_list/). A small difference indicates that the initial CME speed is steady and more reliable.
- (3) $V_{\text{CME}} > V_{\text{ICME}}$, where V_{ICME} is the ICME speed at 1 AU. We do not discuss the events with $V_{\text{CME}} < V_{\text{ICME}}$, because their number is only eight under the limits of the former two conditions.

Figure 1 shows the scatter plots V_{CME} vs. ΔT and V_{CME} vs. V_{ICME} , where $\Delta T = T_{\text{ICME}} - T_{\text{CME}}$. The solid lines are the corresponding quadratic fits whose rms errors (ϵ_{rms}) are 13.4 hour and 127.6 km s^{-1} . The large value of ϵ_{rms} means a bad prediction level in any model if it only uses the value of V_{CME} as an input.

3. Prediction Model

A typical formula for the aerodynamic drag is $f = \frac{1}{2}C\rho S(v - V_{\text{W}})^2$, where C is the dimensionless drag coefficient, ρ , S , v , V_{W} are the ambient density, the cross section, the speed of the body and the wind speed, respectively. Generally speaking, CMEs start to come into a radial expansion phase when they reach a distance r of several R_s away from the sun. The cross section S would increase at a rate of r^2 . The density ρ of its ambient solar wind decreases at a rate of r^{-2} . If we suppose that at CME growth phase its mass and size are in constant proportion, the deceleration of a CME faster than the solar wind can be expressed by (*e.g.*, Cargill, 2004; Vršnak and Žic, 2007):

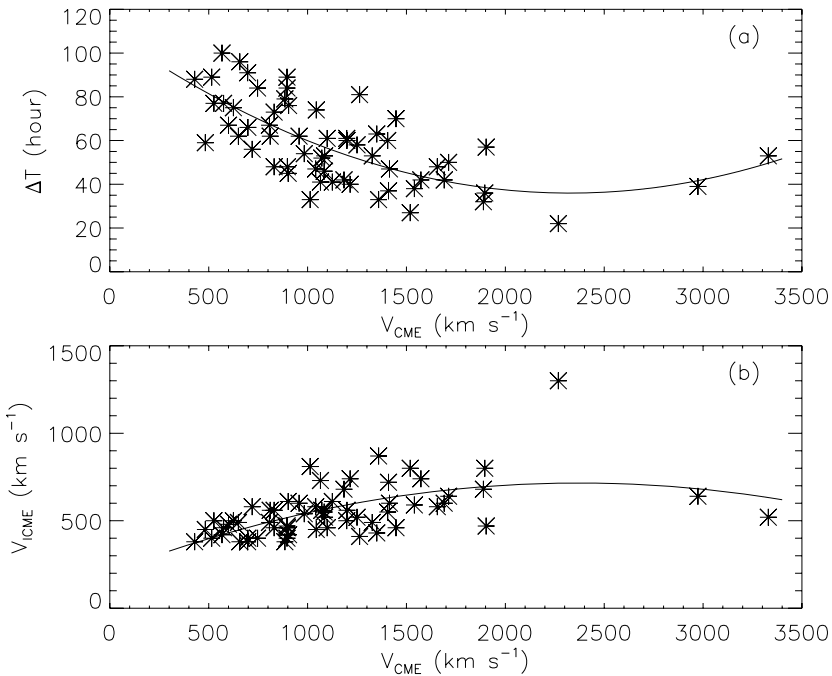
$$\dot{v} = -\frac{1}{A}(v - V_{\text{SW}})^2, \quad (1)$$

Table 1 The list of ICME – CME pairs (units: hour and km s^{-1}).

No.	T_{ICME}	V_{ICME}	$T_{\text{CME}}^{\text{C2}}$	V'_{CME}	T_{CME}	V_{CME}	ΔT	V_{SW}
1	97/04/11 06	470	04/07 1427	878	18	896	84	290
2	10/27 00	500	10/23 1126	503	19	526	77	290
3	11/07 04	400	11/04 0610	785	10	698	66	320
4	98/01/07 01	400	01/02 2328	438	08	515	89	300
5	01/21 06	380	01/17 0409	350	14	429	88	280
6	01/29 20	380	01/25 1526	693	20	657	96	360
7	05/02 05	520	04/29 1658	1374	19	1250	58	320
8	11/09 01	450	11/05 2044	1118	23	1044	74	340
9	00/01/22 17	380	01/18 1754	739	22	697	91	320
10	02/11 16	420	02/08 0930	1079	12	904	76	370
11	02/12 12	540	02/10 0230	944	06	983	54	400
12	02/14 12	520	02/12 0431	1107	07	1086	53	320
13	04/07 06	560	04/04 1632	1188	18	1199	60	350
14	06/08 12	610	06/06 1554	1119	19	1125	41	460
15	07/13 13	610	07/11 1327	1078	16	902	45	340
16	08/12 05	580	08/09 1630	702	21	720	56	430
17	10/05 13	450	10/02 2026	569	02	483	59	360
18	10/13 16	400	10/09 2350	798	04	747	84	320
19	10/28 21	380	10/25 0826	770	14	885	79	310
20	01/03/28 17	480	03/25 1706	677	22	600	67	410
21	04/01 04	600	03/29 1026	942	14	957	62	420
22	04/08 14	740	04/06 1930	1270	22	1215	40	430
23	04/11 22	640	04/10 0530	2411	07	2974	39	520
24	04/13 09	730	04/11 1331	1103	16	1065	41	520
25	04/28 14	550	04/26 1230	1006	16	1084	46	400
26	08/17 20	500	08/14 1601	618	17	625	75	330
27	08/28 00	490	08/25 1650	1433	19	1327	53	370
28	10/01 08	490	09/28 0854	846	13	807	67	450
29	10/12 04	560	10/09 1130	973	14	811	62	340
30	10/21 20	460	10/19 1650	901	20	898	48	310
31	11/06 12	600	11/04 1635	1810	18	1691	42	310
32	11/19 22	430	11/17 0530	1379	07	1350	63	360
33	11/24 14	720	11/22 2330	1437	01	1409	37	400
34	02/04/20 00	500	04/17 0826	1240	11	1198	61	390
35	05/20 10	420	05/16 0050	600	06	567	100	370
36	05/23 20	590	05/22 0326	1557	06	1540	38	360
37	07/18 12	460	07/15 2030	1151	23	1099	61	310
38	08/19 12	460	08/16 1230	1585	14	1447	70	390
39	09/08 04	470	09/05 1654	1748	19	1903	57	400
40	03/05/30 02	600	05/28 0050	1366	03	1414	47	440
41	05/30 22	680	05/29 0127	1237	04	1184	42	440
42	10/29 11	1300	10/28 1130	2459	13	2268	22	510
43	10/31 02	800	10/29 2054	2029	23	1519	27	510
44	11/20 10	580	11/18 0850	1660	10	1656	48	520

Table 1 (Continued.)

No.	T_{ICME}	V_{ICME}	$T_{\text{CME}}^{\text{C2}}$	V'_{CME}	T_{CME}	V_{CME}	ΔT	V_{SW}
45	04/01/22 08	560	01/20 0006	965	04	1072	52	350
46	01/23 23	490	01/21 0454	762	09	650	62	350
47	07/22 18	560	07/20 1331	710	18	831	48	370
48	07/27 02	870	07/25 1454	1333	17	1359	33	430
49	09/14 15	550	09/12 0036	1328	03	1405	60	240
50	11/09 20	640	11/7 1654	1759	18	1713	50	320
51	11/12 08	520	11/10 0226	3387	03	3330	53	390
52	05/01/08 21	460	01/5 1530	735	20	831	73	400
53	01/18 23	800	01/17 0930	2094	11	1896	36	510
54	01/21 19	810	01/20 0654	882	10	1013	33	330
55	02/20 12	410	02/17 0006	1135	03	1263	81	350
56	05/30 01	460	05/26 1506	586	20	575	77	360
57	09/15 06	680	09/13 2000	1866	22	1889	32	510
58	06/08/20 13	400	08/16 1630	888	20	896	89	300
59	12/14 22	740	12/13 0254	1774	04	1573	42	560
60	12/17 00	580	12/14 2230	1042	01	1041	47	560

**Figure 1** The scatter plots V_{CME} vs. ΔT and V_{CME} vs. V_{ICME} . The solid lines are their quadratic fits to the data points.

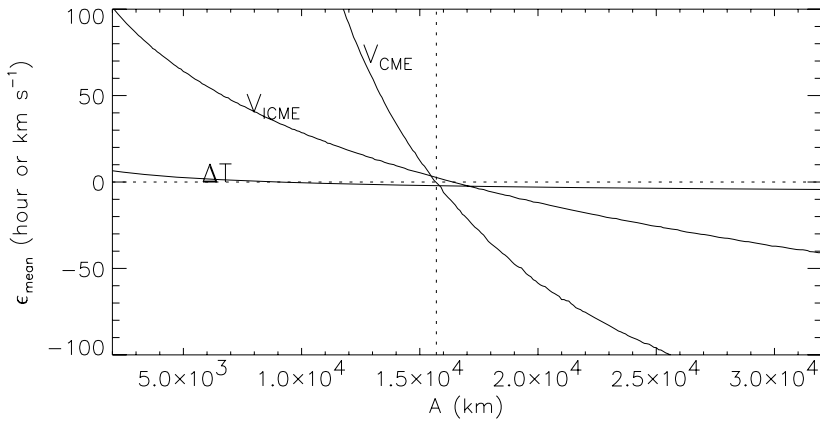


Figure 2 The value of ϵ_{mean} vs. A .

where V_{SW} is the solar wind speed. Such a form completely follows that of the aerodynamic drag force. Grall *et al.* (1996) reported that the solar wind acceleration ceased within a distance of $10R_s$. Sheeley *et al.* (1997) also showed that the acceleration of the solar wind got much smaller at $r > 20R_s$. Therefore, it is acceptable that we assume V_{SW} to be invariable during the whole transit time of a CME.

Integrating Equation (1) we find the following two equations:

$$\frac{\Delta T}{A} = \frac{1}{V_{\text{ICME}} - V_{\text{SW}}} - \frac{1}{V_{\text{CME}} - V_{\text{SW}}}, \quad (2)$$

$$L = V_{\text{SW}}\Delta T + A \ln \frac{V_{\text{CME}} - V_{\text{SW}}}{V_{\text{ICME}} - V_{\text{SW}}}, \quad (3)$$

where $L = 1\text{AU} - 20R_s = 1.36 \times 10^8$ km. Therefore, for the four parameters V_{CME} , V_{ICME} , ΔT and V_{SW} , if we know any two of them, the other two can be calculated. Firstly we should determine the coefficient A . We choose two parameters in $\{V_{\text{CME}}, V_{\text{ICME}}, \Delta T\}$ and change A from 2000 km to 32 000 km, use the above equations to calculate the third parameter, and then compare it with the observed value. As a result, we find that the rms errors (ϵ_{rms}) of ΔT and V_{ICME} change little; they are in the range of 7.8–10.1 hours and 117.8–163.5 km s^{-1} , respectively. The rms error of V_{CME} also changes little when $A > 12000$ km; it changes from 678.8 km s^{-1} to 508.7 km s^{-1} . However, we find that the mean values of error (ϵ_{mean}) change significantly, particularly for V_{CME} and V_{ICME} , as shown in Figure 2. Because of a small difference of A s for $\epsilon_{\text{mean}}(V_{\text{CME}}) = 0$ and $\epsilon_{\text{mean}}(V_{\text{ICME}}) = 0$, we set $A = 15700$, which satisfies $\epsilon_{\text{mean}}(V_{\text{CME}}) = 0$.

Figure 3 depicts the predicted versus observed values of parameters when $A = 15700$ and any two values of V_{CME} , V_{ICME} , ΔT are known. To determine the reliability in prediction, we calculate the cross correlation coefficient (CC) and the values of ϵ_{rms} and ϵ_{mean} . The results are listed as Case 1 in Table 2. Here we use the F -test to estimate the significance level of cross correlation. Its related function is $R_\alpha = \sqrt{\frac{F_\alpha(1, N-2)}{(N-2) + F_\alpha(1, N-2)}}$, where $F_\alpha(1, N-2)$ can be found in a table for the F -test at a given significance level ($\alpha = 0.01$) and the event number ($N = 60$). Only if $\text{CC} \geq R_\alpha = 0.33$, we can say that the correlation is significant. Compared with the last row of Case 3 in Table 2, CC and ϵ_{rms} of ΔT and CC of V_{ICME} are

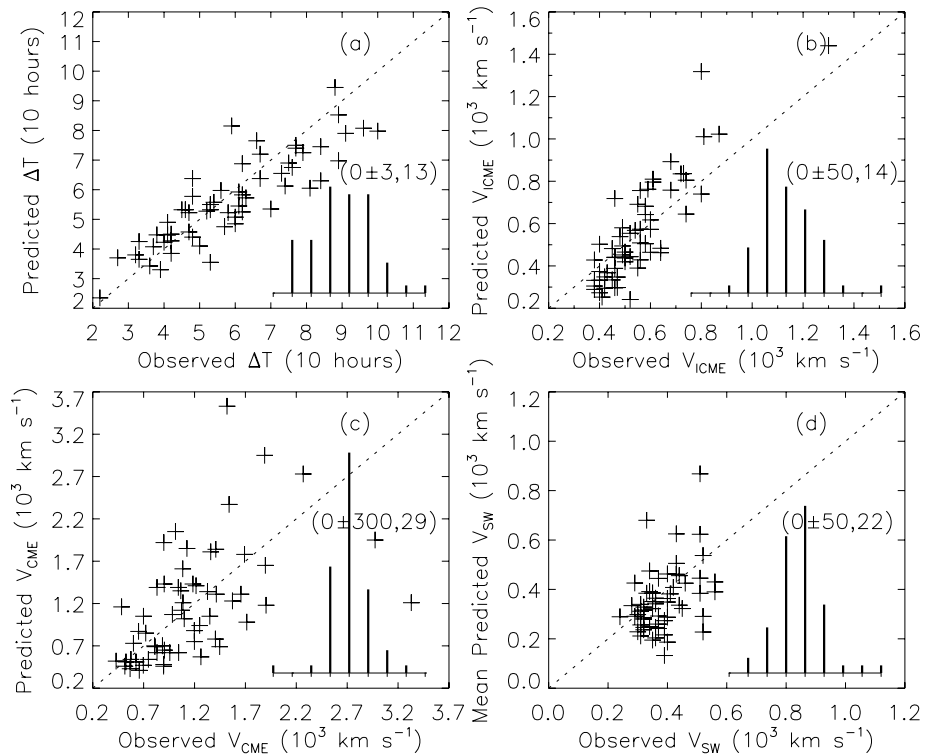


Figure 3 The predicted versus observed values of parameters when we know any two values of $\{V_{CME}, V_{ICME}, \Delta T\}$. The mean predicted V_{SW} in (d) indicates the mean value of three predicted V_{SW} when any two values of $\{V_{CME}, V_{ICME}, \Delta T\}$ are known. The values in brackets near the center of histograms represent the statistics range and the total event number.

Table 2 Four typical parameters used in the model. ‘O’ means the observed value. The values in brackets represent CC, ϵ_{rms} , and ϵ_{mean} , respectively. The last row of Case 3 is for the quadratic fits in Figure 1.

Case	V_{CME} (km s ⁻¹)	V_{ICME} (km s ⁻¹)	ΔT (hour)	V_{SW} (km s ⁻¹)
1	O	O	(0.87, 9.3, -2.2)	(0.40, 110.8, -37.7)
	O	(0.86, 137.5, 2.6)	O	(0.35, 208.9, -19.0)
	(0.53, 594.5, -0.4)	O	O	(0.31, 102.5, -32.2)
2	O	(0.99, 24.1, 7.7)	(0.89, 8.5, -2.4)	$\bar{V}_{SW}$
	(0.98, 117.2, -26.5)	O	(0.80, 11.3, -1.5)	$\bar{V}_{SW}$
	(0.82, 319.4, -44.2)	(0.95, 67.4, 5.3)	O	$\bar{V}_{SW}$
3	O	(0.52, 133.3, 9.0)	(0.67, 13.8, -3.8)	350
	O	(0.65, 126.6, 47.4)	(0.72, 13.0, -4.9)	O
	O	(0.57, 127.6, 0.0)	(0.69, 13.4, 0.0)	—

much improved. However, the prediction for V_{SW} is not very good, because three CC values are close to R_α . From the measurement with the WIND satellite, we know that V_{SW} always fluctuates fast. In this investigation, we put provisionally the observed V_{SW} at the lowest and smooth part of the WIND measurements during several days near T_{ICME} , as given in Table 1.

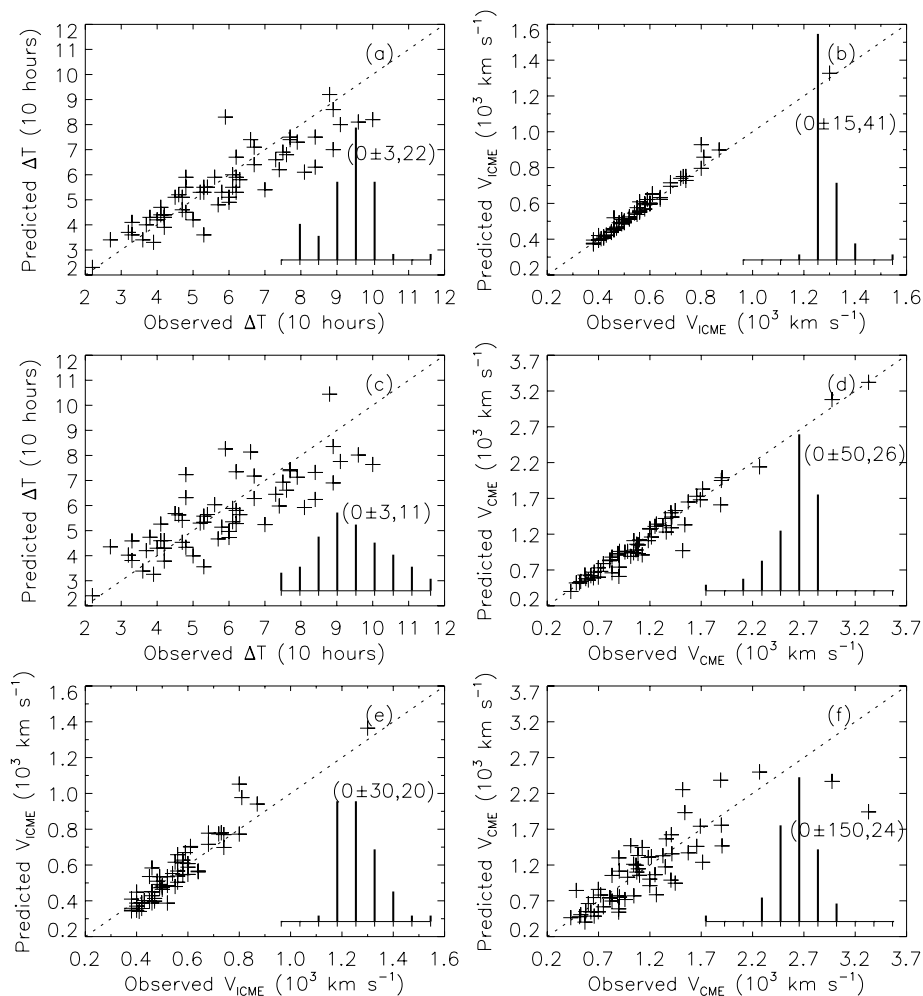


Figure 4 The predicted versus observed values of parameters if we use the value of $\bar{V}_{SW}$ (see Figure 3(d)) and another one parameter of V_{CME} , V_{ICME} or ΔT .

In order to further test our analytical model, we choose one parameter in $\{V_{CME}, V_{ICME}, \Delta T\}$, set V_{SW} to the mean value ($\bar{V}_{SW}$, see Figure 3(d)) of three predicted values of V_{SW} when we know any two values of $\{V_{CME}, V_{ICME}, \Delta T\}$, and estimate the prediction of the other two parameters. The results are shown in Figure 4 and listed as Case 2 in Table 2. We find that all of the predicted and observed values correlate very well, *e.g.*, when the inputs are V_{CME} and $\bar{V}_{SW}$, the values of CC and ϵ_{rms} of ΔT and V_{ICME} are 0.89, 8.5 hour and 0.99, 24.1 km s^{-1} , respectively. Such good correlations indicate that our model is highly consistent with the observations.

In real applications, V_{CME} can be obtained from the SOHO/LASCO movie. V_{SW} can be referred to interplanetary observations or predicted by existing methods (*e.g.*, Arge and Pizzo, 2000; Vršnak, Temmer, and Veronig, 2007). For comparing with those models with a simple input of V_{CME} , we set $V_{SW} \equiv 350 \text{ km s}^{-1}$. The obtained result is shown in Figures 5(a)

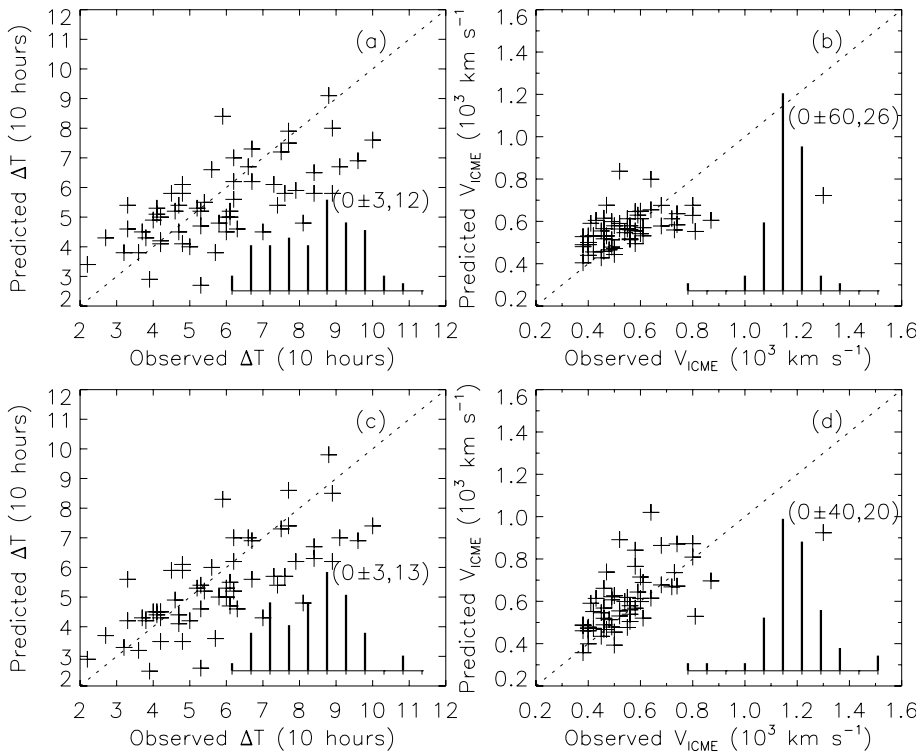


Figure 5 The predicted versus observed values of parameters: (a, b) for $V_{\text{SW}} \equiv 350 \text{ km s}^{-1}$; (c, d) for V_{SW} observed by the WIND satellite.

and (b). The CC and ϵ_{rms} values of ΔT and V_{ICME} are 0.67, 13.8 hour and 0.52, 133.3 km s^{-1} , respectively. It is only a bit worse than their best quadratic fits. On the other hand, if we set V_{SW} from the measurements with the WIND satellite, the precision of the prediction gets a bit better than the quadratic fits, as shown in Figures 5(c) and (d). The above two comparisons are listed as Case 3 in Table 2. Here we think that a major limitation of our model is the difficulty in determining V_{SW} . Vrřnak and Žic (2007) demonstrated that V_{SW} was a dominant parameter in determining the Sun–Earth transit time.

4. Summary and Discussion

In this investigation we present a relatively simple forecast tool to predict the arrival time of ICMEs referring to the aerodynamic drag force. Using the initial speed of CME and the background solar wind speed, the propagation time can be calculated. Sixty events show that our model can lead to a relatively high prediction. Similarly, Vrřnak and Gopalswamy (2002) used the acceleration formula $\dot{v} = -\gamma(v - V_{\text{SW}})$ and $\dot{v} = -\gamma(v - V_{\text{SW}})|(v - V_{\text{SW}}|$ instead of our Equation (1). It is different from ours in that the coefficient γ and the parameter V_{SW} are both variable with r . The good consistency with observations of their and our models indicates that the drag acceleration is really an essential feature of the interplanetary motion of CMEs. Watari (2002) gave a forecast method also using the initial speed and V_{SW}

as inputs based on v having an empirical relation with r . In his model the value of V_{SW} was assumed to be constant at any place between the Sun and the Earth just like in ours. He got $\epsilon_{rms}(\Delta T) = 11.8$ hours in 28 shock events.

There are many other factors which can affect the arrival time of CMEs, as pointed out very recently by Feng *et al.* (2009b). Wei and Dryer (1991) found that the flare-associated shock waves tended to propagate toward the low latitude region and suggested an explanation using the dynamic action of near-Sun magnetic forces. Similarly, Wang *et al.* (2004) also studied the deflection of CME trajectories in the interplanetary medium. Feng and Zhao (2006a) empirically studied the geoeffectiveness of arrival time and geomagnetic storm index Dst for CMEs by considering the effect of heliospheric current sheets. Feng and Zhao (2006b) considered the combined effects of the duration of X-ray flare, the initial shock speed and the total energy of the transient event. The HAF model (Hakamada and Akasofu, 1982; Akasofu, Hakamada, and Fry, 1983; Sun *et al.*, 1985; Fry *et al.* 2001, 2003; McKenna-Lawlor *et al.*, 2008; Smith *et al.*, 2009) may give a global picture of multiple and interacting interplanetary shocks whose inputs include information on the structure of both the solar wind and the interplanetary magnetic field. The ENLIL 3D cone model provides a very clear picture for describing the whole transit process of solar storms (Taktakishvili *et al.*, 2009). Presently, analytical models and numerical MHD-based models are equally important, although they both have limited forecasting possibilities. The analytical models, including some empirical ones, not only provide the forecasting functionality, but also shed some light on devising the numerical MHD-based ones.

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