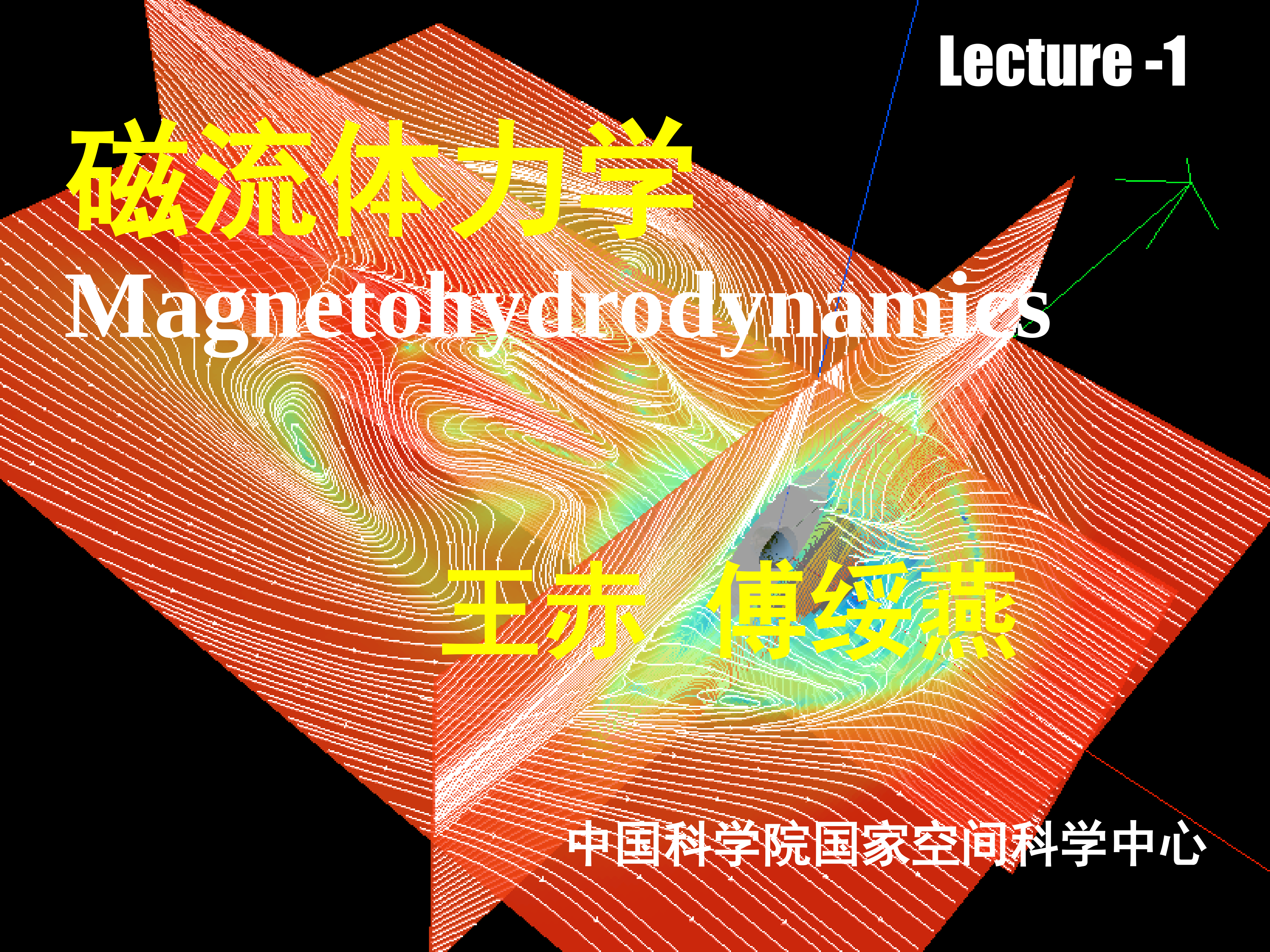

Lecture -1

磁流体力学

Magnetohydrodynamics

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课程安排

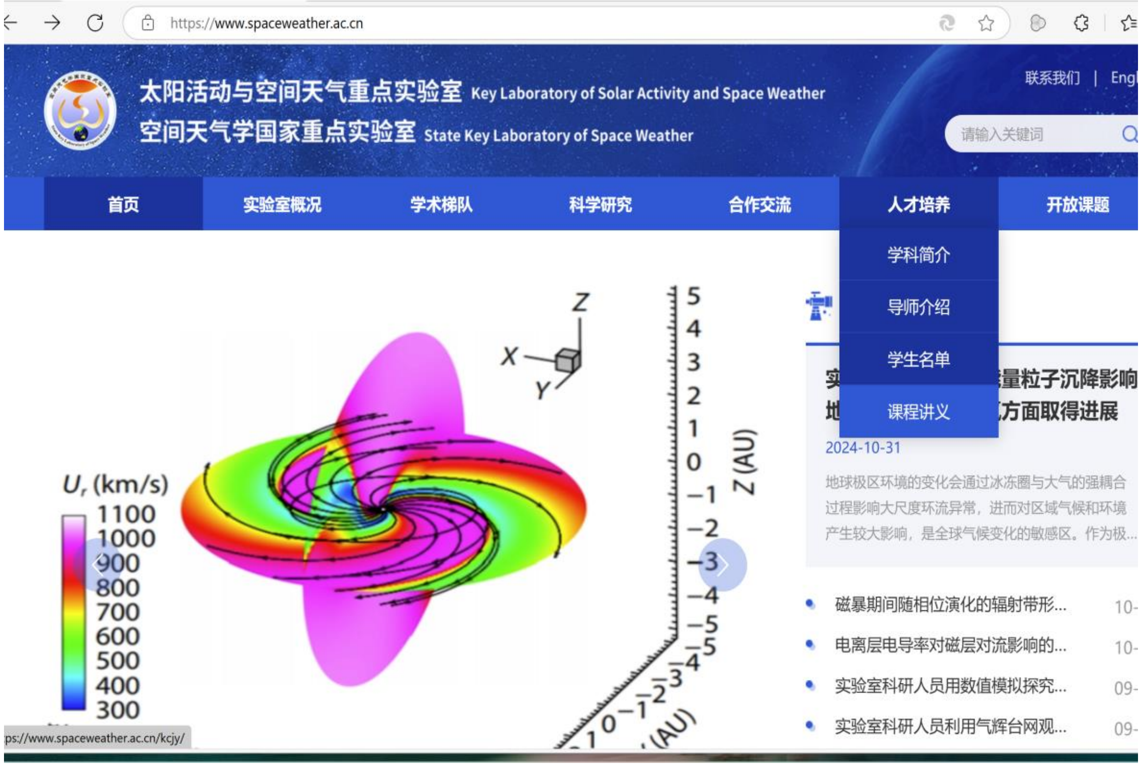
课程	时间	内容
1	11.05	等离子体的流体近似
2	11.08	磁流体静力学
3	11.15	等离子体的冻结与磁重联
4	11.22	磁流体力学波
5	11.26	激波与间断面
6	12.06	磁流体动力学不稳定性
7	12.13	习题讲解和答疑
8	12.20	期末考试

课程材料

空间天气学国家重点实验室网站
spaceweather.ac.cn

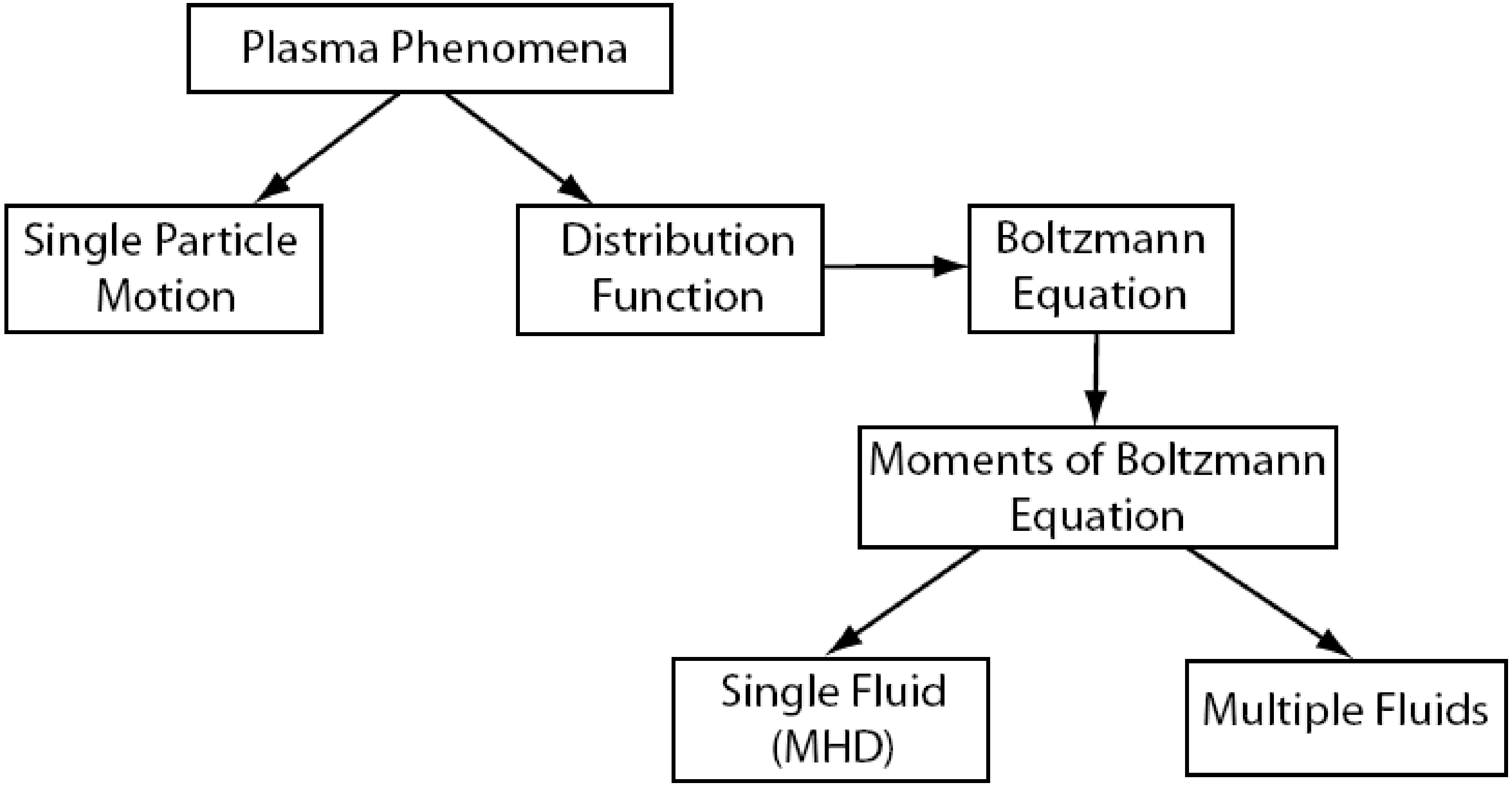
人才培养

课程讲义



平时作业+上课 60%
期末考试40%

Hierarchy of Plasma Models



Fluid Approach

MHD Equations

Fluid Approach (1/2)

- The **E, B** fields are not prescribed but are determined by the positions and motions of the charges themselves.

- A typical plasma density might be 10^{12} ion-electron pairs per cm^3 . If each of these paricles follows a complicated trajectory and it is necessary to follow each of these, predicting the plasma's behavior would be a hopeless task.

- Fortunately, this is not usually necessary, the majority of plasma phenomena observed in real experiments can be explained by a rather crude fluid model.

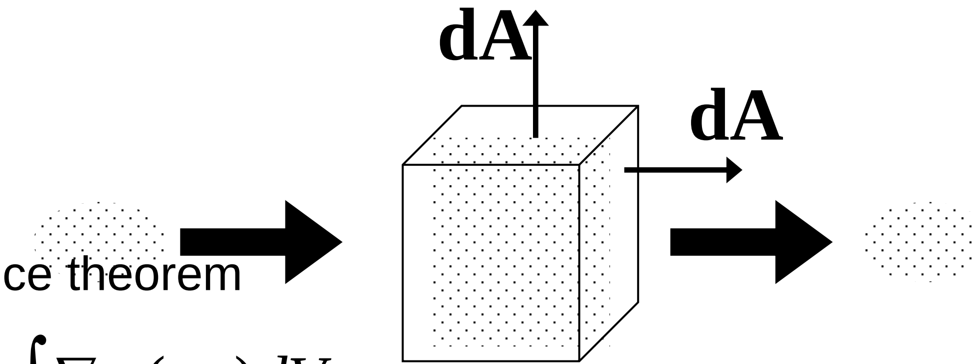
Fluid Approach

- This model is that used in fluid mechanics, in which the indentiy of the individual particle is neglected, and only the motion of fluid elements is taken into account. Of course, the fluid contains electrical charges.

- It is surprising that such a model works for plasmas, which general have infrequent collisions. The ion and electron fluids will interact with each other even in the obsense of collisions, because of the **E** and **B** fields are generated.

Fluid Eq. of Continuity

Conservation of matter requires that the number of particles (N) in a given volume (V) can only change if there is a net particle flux across the surface (S) bounding the Volume (V)



Divergence theorem

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial t}\int_V n dV = -\int_S n\mathbf{u} \bullet d\mathbf{A} = -\int_V \nabla \bullet (n\mathbf{u}) dV$$

Particle flux

Since this must hold for any volume, it means

$$\boxed{\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{u}) = 0}$$

Fluid Eq. of Motion

A. Neglecting collisions and thermal motion

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

If neglect collisions and thermal effects, all particles in a fluid element move together with average velocity u

$$nm\frac{d\mathbf{u}}{dt} = nq(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

d/dt is to be taken at the position of the fluid element, not very convenient. We wish to have equation for fluid elements fixed in space.

Transform to a fixed frame: (convective derivative)

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + (\mathbf{u} \bullet \nabla) G$$

In a plasma with the fluid velocity **u,** we can have:

$$nm[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \bullet \nabla)\mathbf{u}] = nq(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

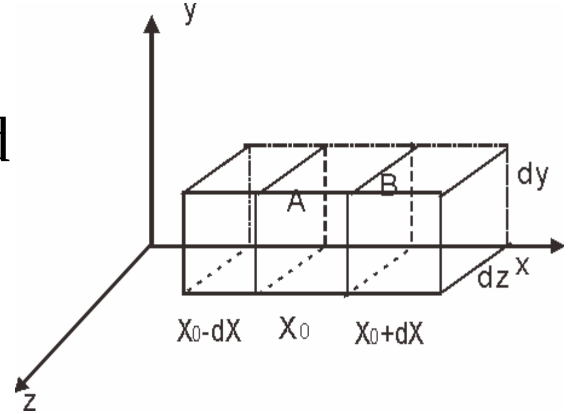
B. Including Thermal Effect (Pressure)

Thermal motion

=➔ the random motion of particles in and out of a fluid element

==➔a pressure force should be added in the eq. of motion for a fluid element

(1) Consider only the x-component of motion though faces A and B of the fluid element centered at (X_0, 1/2dy, 1/2dz).



The number of particles per volume with velocity Vx

$$\Delta n_v = \Delta v_x \iint f(v_x, v_y, v_z) dv_y dv_z$$

The number of particles per second passing through the face A with velocity Vx is

$$N = \Delta n_v v_x \Delta y \Delta z$$

Each particle carries a momentum. The momentum though face A, from particles with Vx>0, in the fluid centered at $(x_0-\Delta x, \frac{1}{2}\Delta y, \frac{1}{2}\Delta z)$

$$P_{A^+} = m\Delta y\Delta z\int_0^\infty v_x^2 f\,dv_x \iint dv_y dv_z$$

$$n = \iiint f dv_x dv_y dv_z \qquad \left\langle v_x^2 \right\rangle = \frac{\int_{-\infty}^{\infty} v_x^2 f dv_x \iint dv_y dv_z}{\iiint f dv_x dv_y dv_z}$$

(2) It can be written as:

$$P_{A^+} = m\Delta y\Delta z\frac{1}{2}\left[n\left\langle v_x^2\right\rangle\right]_{x_0-\Delta x}$$

The factor ½ comes from the fact that only half the particles in the cube are going toward A.

Similarly, the momentum carried out though face B is

$$P_{B^+} = m\Delta y\Delta z\frac{1}{2}\left[n\left\langle v_x^2\right\rangle\right]_{x_0}$$

(3) The net gain in momentum for particles:

$$P_{A^+} - P_{B^+} = m\Delta y\Delta z\frac{1}{2}\left[(n\left\langle v_x^2\right\rangle)_{x_0-\Delta x} - (n\left\langle v_x^2\right\rangle)_{x_0}\right]$$

$$= \frac{1}{2}m\Delta y\Delta z(-\Delta x)\frac{\partial}{\partial x}(n\left\langle v_x^2\right\rangle)$$

This results is doubled by the contribution of left-moving particles, since they carry x momentum and also move in the opposite direction relative to the gradient. $n\left\langle v_x^2\right\rangle$

The total change of momentum of the fluid element at X_0 is therefore

$$\frac{\partial}{\partial t}(nmu_x)\Delta x\Delta y\Delta z = -m\frac{\partial}{\partial x}(n\left\langle v_x^2\right\rangle)\Delta x\Delta y\Delta z$$

(4) Let the velocity of a particle be composed by fluid velocity plus random thermal motion.

$$v_x = u_x + v_{xr}$$

For a 1D Maxwillian distribution, $\frac{1}{2} m \left\langle v_{xr}^2 \right\rangle = \frac{1}{2} kT$

$$\left\langle v_x^2 \right\rangle = u_x^2 + kT/m$$

then
$$\frac{\partial}{\partial t}(nmu_x) = -m \frac{\partial}{\partial x}[n(u_x^2 + kT/m)]$$

re-group
$$nm(\frac{\partial}{\partial t} u_x + u_x \frac{\partial}{\partial x} u_x) + mu_x(\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} nu_x) = -\frac{\partial}{\partial x}(nkT)$$

Continuity
$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} nu_x = 0$$

$$\boxed{nm(\frac{\partial}{\partial t} u_x + u_x \frac{\partial}{\partial x} u_x) = -\frac{\partial}{\partial x} p}$$

Generalizing to three dimensions, we have

$$nm\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right] = -\nabla p$$

Add the pressure-gradient force with the electromagnetic forces:

$$nm[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \bullet \nabla)\mathbf{u}] = nq(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla \bullet \mathbf{P}$$

What we have derived is only a special case: the transfer of x momentum by motion in the x direction; and we have assumed that the fluid is isotropic, so that the same result holds in the y and z directions.

But it is possible to transfer y momentum by motion in the x direction. Suppose that the y-velocities of particles at x_0-Δx and x_0+ Δx were a maximum, and
that vy = 0 at x_0. Then particles passing through Faces A and B would bring more y-momentum into the fluid element at x0 than they take out. This would give rise to a shear stress on the fluid element at x_0, which must be described in general by a stress tensor, **P,** The off-diagonal elements of P are usually associated with viscosity.

C. Including Collisions

If a neutral gas is present, the charged fluid can exchange momentum with it through collisions. The momentum lost per collision will be proportional to the relative velocity between the changed fluid and the neutral gas (u-u_0), where u_0 is the velocity of the neutral fluid. If τ , the mean free time between collisions, is approximately constant, the resulting force term can be roughly written as $-nm(\boldsymbol{u}-\boldsymbol{u}_0)/\tau$ or $-\sigma nm(\boldsymbol{u}-\boldsymbol{u}_0)$,where σ is the collision frequency.

$$nm[\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\bullet\nabla)\mathbf{u}]=nq(\mathbf{E}+\mathbf{u}\times\mathbf{B})-\nabla\bullet\mathbf{P}-\sigma mn(\mathbf{u}-\mathbf{u}_0)$$

(A) When we took the velocity distribution to be Maxwellian, and therefore the averages of velocity, we assumed that there were collisions. However, the fluid theory is not sensitive to the distribution function, as long as we can use the same average velocity.

(B) The other reason for the fluid theory to work for the plasma is that magnetic field and wave particle interaction may play the role as collisions in a certain sense.

Fluid Eq. of State

We use the thermodynamic equation of state to close the equations.

$$p = C\rho^{\gamma}$$

$$\gamma = C_p / C_v$$

$$\text{===}\!\rightarrow \quad \frac{\nabla p}{p} = \gamma \frac{\nabla \rho}{\rho} = \gamma \frac{\nabla n}{n}$$

$$\boxed{p_j n_j^{-\gamma} = const}$$

Fluid Eq. of State

(1) For isothermal compression: $\nabla p = kT\nabla n \Rightarrow \gamma = 1$

(2) Adiabatic compression (T also changes)

$$\frac{\nabla n}{n} + \frac{\nabla T}{T} = \gamma \frac{\nabla n}{n} \Rightarrow \frac{\nabla T}{T} = (\gamma - 1)\frac{\nabla n}{n}$$

(3) More general (adiabatic), $\gamma = (2+N)/N$

where N is the number of degrees of freedom, it is valid for negligible heat flow.

Isotropic plasma and Anisotropic plasma

- A plasma is called isotropic, if its pressure tensor is diagonal with all diagonal elements having the same value $P = p\,I$

- Particle distribution in a plasma often anisotropic.

- Magnetized plasma: particle dynamics parallel to the field direction ($\vec{B} \parallel \vec{e}_z$) different from the perpendicular particle dynamics.

$$\Rightarrow \quad \mathcal{P} = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_{||} \end{pmatrix} .$$

- Both pressures related to a kinetic temperature by the ideal gas law

$$p_{||} = nk_B T_{||}$$

$$p_\perp = nk_B T_\perp$$

MultiFluid Equation:

$$\rho = n_e q_e + n_i q_i = e(-n_e + Z n_i)$$

$$\mathbf{j} = n_e q_e \mathbf{v}_e + n_i q_i \mathbf{v}_i = e(-n_e \mathbf{v}_e + Z n_i \mathbf{v}_i) = -e n_e (\mathbf{v}_e - \mathbf{v}_i)$$

$$\frac{\partial n_j}{\partial t} + \nabla \bullet (n_j \mathbf{v}_j) = 0$$

$$n_j m_j [\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \bullet \nabla)\mathbf{v}_j] = n_j q_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j - \sigma_{jk} n_j m_j (\mathbf{v}_j - \mathbf{v}_k)$$

$$p_j n_j^{-\gamma} = const$$

Poisson's equation

$$\nabla \bullet \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \bullet \mathbf{B} = 0$$

Ampere's law

Maxwell's Equations

Faraday's law

Accounting

Unknows		Equations	
n_e, n	2	Continity e,i	2
$\boldsymbol{\upsilon}_e, \boldsymbol{\upsilon}_i$	6	Momentum e,i	6
p_e, p_i	2	State e,i	2
E, **B**	6	Maxwell	8
	16		18

But 2 of Maxwell ($\nabla \cdot$ equs) are redundant because can be deduced from others: e.g.

$$\nabla \cdot (\nabla \times \mathbf{E}) = 0 = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{B})$$

and

$$\nabla \cdot (\nabla \times \mathbf{B}) = 0 = \mu_0 \nabla \cdot \mathbf{j} + \frac{1}{c^2} \frac{\partial(\nabla \cdot \mathbf{E})}{\partial t}$$

so 16 equs for 16 unknowns.

$$\nabla \cdot \boldsymbol{j} + \frac{\partial \rho}{\partial t} = 0$$

Magnetohydrodynamics (MHD)

- Description of plasma as a multi-fluid system is often too complicated.

  Simplified to single fluid description (MHD)

- The MHD model is applicable only when charge separation (e.g. plasma oscillations or electromagnetic waves in plasmas) is negligible. The condition for it is that the length scales should be larger than the Debye length and the time scales larger than the inverse of plasma frequency.

Assuming that plasma consists of electrons (m_e, q_e=-e) and one component of ions (m_i, q_i=e)

Single-Fluid approximation

Mass density: $\rho_m = n_e m_e + n_i m_i$

Velocity: $\mathbf{V} = (n_e m_e \mathbf{v}_e + n_i m_i \mathbf{v}_i)/(n_e m_e + n_i m_i)$

Charge density: $\rho_q = n_e q_e + n_i q_i$

Current density: $\mathbf{j} = q_e n_e \mathbf{v}_e + q_i n_i \mathbf{v}_i = q_e n_e (\mathbf{v}_e - \mathbf{v}_i)$

Total pressure: $p = p_i + p_e$

$m_e(Ce) + m_i(Ci)$

$$m_e \frac{\partial n_e}{\partial t} + m_e \nabla \bullet (n_e \mathbf{v}_e) = 0 \qquad m_i \frac{\partial n_i}{\partial t} + m_i \nabla \bullet (n_i \mathbf{v}_i) = 0$$

$$\frac{\partial (m_i n_i + m_e n_e)}{\partial t} + \nabla \bullet (m_i n_i \mathbf{v}_i + m_e n_e \mathbf{v}_e)$$

$$= \frac{\partial (m_i n_i + m_e n_e)}{\partial t} + \nabla \bullet [(m_i n_i + m_e n_e) \frac{m_i n_i \mathbf{v}_i + m_e n_e \mathbf{v}_e}{m_i n_i + m_e n_e}]$$

Mass Conservation

$$\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m \mathbf{V}) = 0$$

$q_e(Ce) + q_i(Ci)$

$$q_e \frac{\partial n_e}{\partial t} + q_e \nabla \bullet (n_e \mathbf{v}_e) = 0$$

$$q_i \frac{\partial n_i}{\partial t} + q_i \nabla \bullet (n_i \mathbf{v}_i) = 0$$

$$\Downarrow$$

Charge Conservation

$$\frac{\partial \rho_q}{\partial t} + \nabla \bullet \mathbf{j} = 0$$

Nssc

(Me) + (Mi)

$$n_j m_j [\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \bullet \nabla)\mathbf{v}_j] = n_j q_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j - \sigma_{jk} n_j m_j (\mathbf{v}_j - \mathbf{v}_k)$$

RHS:====➔

$$\sum_j \{n_j q_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j + \mathbf{F}_{jk}\} = \rho_q \mathbf{E} + \mathbf{j} \times \mathbf{B} - \nabla(p_e + p_i)$$

($\mathbf{F}_{ei}$ = -$\mathbf{F}_{ie}$, so no net friction)

LHS==➔: $\sum_j m_j n_j \, [\frac{\partial}{\partial t} + \mathbf{v}_j \bullet \nabla] \, \mathbf{v}_j$

(1) ignore the contribution of electron momentum ($m_e << m_i$)

(2) V is approximated as v_i

$$\sum_j m_j n_j \, [\frac{\partial}{\partial t} + \mathbf{v}_j \bullet \nabla] \, \mathbf{v}_j \approx \rho_m (\frac{\partial}{\partial t} + \mathbf{V} \bullet \nabla)\mathbf{V}$$

Momentum Eq.

$$\rho_m \, [\frac{\partial}{\partial t} + \mathbf{V} \bullet \nabla] \, \mathbf{V} = \rho_q \mathbf{E} + \mathbf{j} \times \mathbf{B} - \nabla p$$

$$\frac{q_e}{m_e}(Me) + \frac{q_i}{m_i}(Mi)$$

$$n_j m_j[\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \bullet \nabla)\mathbf{v}_j] = n_j q_j(\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j - \sigma_{jk} n_j m_j(\mathbf{v}_j - \mathbf{v}_k)$$

$$\sum_j n_j q_j[\tfrac{\partial}{\partial t} + \mathbf{v}_j \bullet \nabla]\mathbf{v}_j = \sum_j [\frac{n_j q_j{}^2}{m_j}(\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \frac{q_j}{m_j}\nabla p_j + \frac{q_j}{m_j}\mathbf{F}_{jk}]$$

$$\rho_m = n_e m_e + n_i m_i \qquad \mathbf{j} = n_e q_e \mathbf{v}_e + n_i q_i \mathbf{v}_i = n_e q_e (\mathbf{v}_e - \mathbf{v}_i)$$

$$\sum_j n_j q_j [\tfrac{\partial}{\partial t} + \mathbf{v}_j \bullet \nabla]\mathbf{v}_j \approx \sum_j n_j q_j [\tfrac{\partial}{\partial t}\mathbf{v}_j] = n_i q_i \frac{\partial \mathbf{v}_i}{\partial t} + n_e q_e \frac{\partial \mathbf{v}_e}{\partial t}$$

For simplicity, we treat v as small and neglect the term $(v \cdot \nabla)v$

$$\rho_m \frac{\partial}{\partial t}(\frac{\mathbf{j}}{\rho_m}) = (n_e m_e + n_i m_i)\frac{\partial}{\partial t}(\frac{n_e q_e \mathbf{v}_e + n_i q_i \mathbf{v}_i}{n_e m_e + n_i m_i})$$

$$= (n_i + \frac{n_e m_e}{m_i})\frac{\partial}{\partial t}\frac{n_e q_e \mathbf{v}_e + n_i q_i \mathbf{v}_i}{n_i + \frac{n_e m_e}{m_i}} \approx n_i \frac{\partial}{\partial t}\frac{n_e q_e \mathbf{v}_e + n_i q_i \mathbf{v}_i}{n_i}$$

LHS==➔: $$\sum_j n_j q_j [\tfrac{\partial}{\partial t} + \mathbf{v}_j \bullet \nabla]\mathbf{v}_j \approx \rho_m \frac{\partial}{\partial t}(\frac{\mathbf{j}}{\rho_m})$$

RHS:====➔

[I]

$$\sum_j \frac{n_j q_j^2}{m_j}\mathbf{E} = n_e^2 q_e^2 \left(\frac{1}{n_e m_e} + \frac{1}{n_i m_i}\right)\mathbf{E}$$

$$= n_e^2 q_e^2 \left(\frac{n_i m_i + n_e m_e}{n_e m_e n_i m_i}\right)\mathbf{E}$$

$$= -\frac{q_e q_i}{m_e m_i}\rho_m \mathbf{E}$$

[II]

Left for excise

$$\sum_j \frac{n_j q_j^2}{m_j}\mathbf{v}_j = n_e q_e^2 \frac{\mathbf{v}_e}{m_e} + n_i q_i^2 \frac{\mathbf{v}_i}{m_i}$$

$$= \frac{q_e q_i}{m_e m_i}\{\frac{n_e q_e m_i}{q_i}\mathbf{v}_e + \frac{n_i q_i m_e}{q_e}\mathbf{v}_i\}$$

$$= -\frac{q_e q_i}{m_e m_i}\{n_i m_i \mathbf{v}_e + n_e m_i \mathbf{v}_i\}$$

$$[n_i q_i = -n_e q_e]$$

$$= -\frac{q_e q_i}{m_e m_i}\{n_e m_e \mathbf{v}_e + n_i m_i \mathbf{v}_i - (\frac{m_i}{q_i} + \frac{m_e}{q_e})(q_e n_e \mathbf{v}_e + q_i n_i \mathbf{v}_i)\}$$

$$= -\frac{q_e q_i}{m_e m_i}\{\rho_m \mathbf{V} - (\frac{m_i}{q_i} + \frac{m_e}{q_e})\mathbf{j}\}$$

[III]

$$\mathbf{F}_{ei} = -\sigma_{ei} n_e m_e (\mathbf{v}_e - \mathbf{v}_i) = -\mathbf{F}_{ie}$$

$$\sum_j \frac{q_j}{m_j} \mathbf{F}_{jk} = -\sigma_{ei} (n_e q_e - n_e q_i \frac{m_e}{m_i})(\mathbf{v}_e - \mathbf{v}_i)$$

$$= -\sigma_{ei} (1 - \frac{q_e m_e}{q_i m_i}) \mathbf{j}$$

LHS=RHS

$$\rho_m \frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{\rho_m}\right) = -\frac{q_e q_i}{m_e m_i}[\rho_m \mathbf{E} + \{\rho_m \mathbf{v} - (\frac{m_i}{q_i} + \frac{m_e}{q_e})\mathbf{j}\} \times \mathbf{B}$$
$$-(\frac{q_e}{m_e}\nabla p_e + \frac{q_i}{m_i}\nabla p_i) - (1 - \frac{q_e m_e}{q_i m_i})\sigma_{ei}\mathbf{j}$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = -\frac{m_e m_i}{q_e q_i}\frac{\partial}{\partial t}(\frac{\mathbf{j}}{\rho_m}) + \frac{1}{\rho_m}(\frac{m_i}{q_i} + \frac{m_e}{q_e})\mathbf{j} \times \mathbf{B}$$
$$-(\frac{q_e}{m_e}\nabla p_e + \frac{q_i}{m_i}\nabla p_i)\frac{m_e m_i}{q_e q_i \rho_m} - (1 - \frac{q_e m_e}{q_i m_i})\frac{m_e m_i}{q_e q_i \rho_m}\sigma_{ei}\mathbf{j}$$

General Ohm's Law

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=-\frac{m_e m_i}{q_e q_i}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{\rho_m}\right)+\frac{1}{\rho_m}\left(\frac{m_i}{q_i}+\frac{m_e}{q_e}\right)\mathbf{j}\times\mathbf{B}$$

$$-\left(\frac{q_e}{m_e}\nabla p_e+\frac{q_i}{m_i}\nabla p_i\right)\frac{m_e m_i}{q_e q_i \rho_m}$$

$$-\left(1-\frac{q_e m_e}{q_i m_i}\right)\frac{m_e m_i}{q_e q_i \rho_m}\sigma_{ei}\mathbf{j}$$

electron inertia, negligible for low enough frequency

~ the resistivity

Hall term

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\eta\mathbf{j}+\frac{1}{ne}\mathbf{j}\times\mathbf{B}-\frac{1}{ne}\nabla\bullet\mathbf{P}_e+\frac{m_e}{ne^2}\frac{\partial\mathbf{j}}{\partial t}$$

General Ohm's Law

Last term in **J** has a coefficient, ignoring m_e/m_i c.f. 1

$$\frac{m_e m_i \overline{v_{ei}}}{q_e q_i (n_i m_i)} = \frac{m_{ei}\overline{v_{ei}}}{q_e^2 n_e} = \eta \qquad (\sigma_{ei} = \overline{v_{ei}})$$

The resistivity

Hence dropping electron inertia, Hall term, pressure term, the Ohm's law becomes:

$$\mathbf{E} + \upsilon \times \mathbf{B} = \eta \mathbf{j}$$

Equation for the State

$$p_e n_e^{-\gamma_e} + p_i n_i^{-\gamma_i} = const.$$

$$\gamma_e = \gamma_i = \gamma$$

$$\rho_m \approx n m_i$$

$$p \rho_m^{-\gamma} = const$$

Summary of MHD equatons (I)

Mass Conservation $$\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m \mathbf{V}) = 0$$

Momentum $$\rho_m \left(\frac{\partial}{\partial t} + \mathbf{V} \bullet \nabla\right)\mathbf{V} = \rho_q \mathbf{E} + \mathbf{j} \times \mathbf{B} - \nabla p$$

Eq. State $$p \rho_m^{-\gamma} = const$$

Charge Conservation $$\frac{\partial \rho_q}{\partial t} + \nabla \bullet \mathbf{j} = 0$$

Ohm's Law $$\mathbf{E} + \upsilon \times \mathbf{B} = \eta \mathbf{j}$$

- Since the fluid is assumed to be electrically neutral, the charge density ρ_q is taken to be zero.

- We shall not need Poisson's equation because that is taken care of by quasi-neutrality

- Normally, one uses MHD only for low frequency phenomena, so the Maxwell displacement current $\frac{1}{c^2}\frac{\partial E}{\partial t}$ can be ignored in comparison with the conduction current.

Summary of MHD Equations (II)

In order to obtain a compete and self-consistent description of a resistive MHD plasma, the fluid equations and Ohm's law must be combined with Maxwell's equations:

Ampere's Law $\nabla \times \boldsymbol{B} = \mu o \boldsymbol{J}$

$$\nabla \bullet \mathbf{B} = 0$$

Faraday's law $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

Gauss's law $\nabla \cdot \mathbf{E} = 0$

Ideal MHD equations

As the collision frequency goes to zero, the condctivity goes to infinity: ideal MHD plasma

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0$$

The fluid velocity component perpendicular to **B** is then give by **E** x **B**/ B^2 , which is identical to the **E** x **B** drift velocity encountered in single particle orbit theory.